SEEING MATHEMATICS THROUGH A NEW LENS

using photos in the mathematics classroom

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Over the last decade, many teachers have embraced the challenge of incorporating open-ended problems in the mathematics classroom. Open-ended problems, compared to closed problems, present students with varied approaches or multiple solutions to a problem. Research suggests that using open-ended problems in the classroom is an effective teaching strategy for establishing, consolidating, extending, reinforcing and reflecting on mathematical concepts (Busatto, 2004). Through open-ended problems students are presented with opportunities to explore varied strategic approaches and encouraged to think flexibly about mathematics. However, for teachers, particularly those more familiar with closed mathematics problems, learning to develop and pose open-ended problems is not a trivial activity and can prove to be a difficult task.

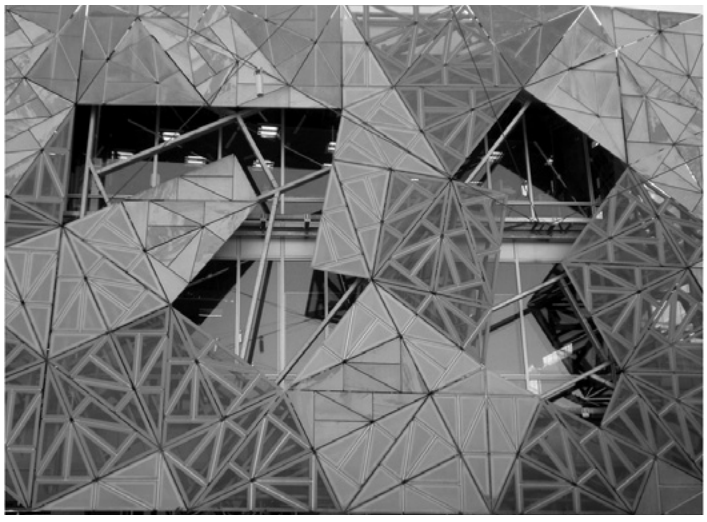
In this article, we present an approach to developing open-ended problems through capturing contextualised mathematics in photographs. We draw upon our research with the Problem Posing Research Project, a collaborative venture between an Australian and a Canadian university to broaden pre-service teachers pedagogical practices in the development of problem posing (see Bragg & Nicol, 2008; Nicol & Bragg, 2009). Based on our research findings and our personal engagement with open-ended problem photos, we argue that while the process of developing open-ended problem photos is not without its challenges, it can ultimately enhance an educator's ability to connect with mathematics in ways that open possibilities for seeing mathematics differently. We contend that through creating open-ended problem photos, teachers and students will see mathematics through a new lens.

One of the aims of the new Australian Curriculum is to ensure that students "are confident, creative users and communicators of mathematics, able to investigate, represent and interpret situations in their personal and

work lives and as active citizens" (Australian Curriculum Assessment and Reporting Authority, 2010). Fostering confident approaches to mathematics in our students' personal and future work lives are challenges facing educators. Students need to see, then understand the connection between the mathematics found in and outside of the classroom, and not view them as separate entities. Our goal is to build students' and teachers' awareness of the beauty and complexity of the mathematics around them. One method for achieving this is through activities that incorporate photography with open-ended problem posing.

An important aspect of open-ended problem photos is that they create a curiosity in the students and a desire to explore possible solutions. Our experience of developing open-ended problem photos has heightened our awareness of the mathematical environment in which we live and has led to a "Math Curse"-like experience. For those who are not familiar with Scieszka's (1995) picture book "Math Curse", the main character's teacher tells the class that he sees almost everything as a mathematical problem. The main character finds she is suddenly in the grip of a math curse and her life is transformed into a succession of mathematics problems. We predict that after reading this paper you too will be infected with the 'math curse' and see your environment as a series of potential mathematical problems.

What are open-ended problem pictures?



- The triangular shapes can combine to create other polygons. Draw and label five of these polygons in your book and the triangles inside the polygons.
- There are many different angles evident on the wall. Name the polygon and the angles they create.
- Identify and name four different polygons with perpendicular lines. Identify and name four polygons with at least one set of parallel lines.

Figure 1. Problem picture photo and questions with a focus on shape created by an Australian preservice teacher.

An open-ended problem picture is a photograph of an object, scene or activity that is accompanied by one or more open-ended mathematical word problems based on the context of the photo. Richard Phillip's Problem Pictures website (www.problempictures.co.uk) offers hundreds of digital images as a source of mathematics problems—both open and closed problems. Alternatively educators and students can collect their own photographic images and design open-ended problems based on these images. Sullivan and Lilburn's (2004) popular educational text, *Open-Ended Maths Activities: Using 'Good' Questions to Enhance Learning in Mathematics*, is a good resource for designing open-ended problems. Collecting images familiar to students

is an ideal way to pique students' interest in open-ended problem photos. Resources such as Sparrow and Swan (2005, p. 2, 114) provide detailed lists of local subject matter that is rich with mathematical potential and ideal for developing open-ended problem photos. For example, the photograph and accompanying open-ended questions in Figure 1 build on Melbourne students' familiarity with a well-known landmark, Federation Square in Melbourne, Australia.

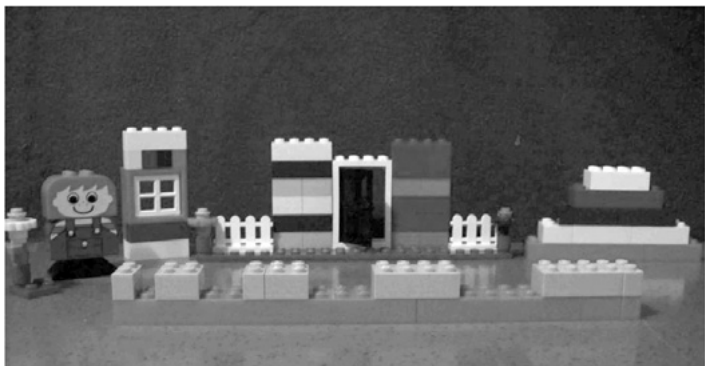
The Problem Posing Research Project examined the responses of 197 preservice teachers taught by Bragg to a course assignment that involved developing open-ended problem photos. The preservice teachers were enrolled in a teacher education program. The data collected included work samples, survey responses, interview data, and research field notes. We were interested particularly in the benefits and challenges of the task, and the impact of the problem posing process in broadening preservice teachers' pedagogical repertoire to incorporate mathematics in the environment.

Designing open-ended problem photos

What is the starting point for creating open-ended problem photos? Findings from the Problem Posing Research Project suggested that two main approaches were used when developing open-ended problem photos:

1. starting with a problem; and,
2. starting with a photo.

By *starting with a problem*, the problem poser begins by considering a mathematical concept and the local curriculum documents and then creates open-ended questions with an image in mind. The fun begins for problem posers using this approach as they search for photos that capture the images that match their questions. Problem posers may find there are many possible photos that will cater for a question. Through this approach the mathematics is at the forefront and drives the selection of the image. Alternatively, the photos may be staged to fit the requirements of the open-ended problem, such as shown in the novel photo of building blocks taken by a Canadian preservice teacher (see Figure 2). The inspired use of resources in the home environment can assist in the development of a series of questions that could stimulate students' interests.



- Describe a pattern you see in Lego World (it can be a vertical up-and-down pattern, or a horizontal side-to-side pattern). What would the next Lego block in your pattern be? Explain your pattern in algebraic terms. Can you find more?
- Lego Lucy has 50 green hedge blocks that are 3 cm long to put around the edge of her yard. What could the length, width and perimeter of her yard be if her house measured 15 cm × 20 cm?

Figure 2. Staged photo with accompanying questions.

A second approach to designing open-ended problem pictures is *starting with a photo*. This approach requires an exploration of the local area while armed with a camera. Immersion in the environment heightens the problem poser's awareness of the potential for

One of the gumballs in this picture equals one gram. What objects in the classroom/your home are larger in size than this gumball but have the same mass? What objects in the classroom/your home are smaller in size than this gumball but have the same mass?

Figure 3. Everyday items employed to explore mathematical concepts.

mathematics in everyday images. This heightened awareness in the environment, as noted by preservice teachers in our research, is a positive outcome of engagement in the open-ended problem photos task. In our research both approaches appeared to be equally successful in resulting in open-ended problems. An analysis of 444 open-ended problem photos created by novice teachers revealed that 97% of the problems were open-ended in nature (Nicol & Bragg, 2009). The process however is not always linear but rather cyclical. Results indicate that the preservice teachers rethink the problems or the photos and undertake both processes when developing open-ended problem photos.

Some everyday objects that offer potential for exploring mathematics are: buildings (angles, lines, symmetry, see Figure 1), traffic signs (shapes), tiles and bricks (tessellation, pattern, see Figure 2), food items at home or in the supermarket (measurement—mass and money, multiplicative thinking, algebra, see Figure 3), sporting events (time, distance, speed, statistics), shop windows (timetables, spatial displays, pricing/ money).

A practical and accessible method for developing open-ended questions is outlined below from Sullivan and Lilburn's text (2004, pp. 5–6).

Method 1: Working backwards

Identify a topic.
Think of a closed question and write down the answer
Make up a new question that includes the answer as part of the question.
e.g. How many chairs are in the room? (4)
becomes…
I counted something in our room. There were exactly four. What might I have counted?

Method 2: Adapting a standard question

Identify a topic.
Write down a complete question including the answer
Adapt it to make an open question.
e.g. What is the time shown on this clock?
becomes…
What is your favourite time of day?
Show it on a clock.

Colleagues can be invited to assist in developing problems based on the photos. Print photos onto a large sheet of paper allowing enough space for

annotated comments. Leave the photos on the staff room table or on a pin board. Teachers are invited to inscribe on the print out questions that could be asked or the different concepts that might be drawn from the photos selected. Figure 4 illustrates the potential range of questions that can be developed from one simple image.

The problems above incorporate 2D and 3D shape, nets, measurement (size, perimeter, area, and time), space featuring design, angles, and pattern. This is a small sample of the possibilities for mathematical exploration within this image.

Photo by Jan Huisken

Open-ended problems

- Describe the shapes in the photo, e.g. number of faces, edges. Categorise into 2D and 3D shapes.
- Make a net for a 3D shape you see in the photo.
- Describe and draw the different patterns you can see in this photo.
- How many children could stand in the visible part of this playground? Explain your working.
- Identify different types of angles and sizes (degrees/ amount of turn).
- Estimate the size of objects visible in the photo.
- What time of day is this photo taken? How did you arrive at your answer?
- We can only see a quarter of this playground in the picture. Describe a way to measure the area of this entire playground. How could you work out the area using the objects in the photo?
- If this is a quarter of this playground, what are the possible perimeters of this playground?

Figure 4. Photo of playground with accompanying open-ended problems.

Interactive versus illustrative problem photos

In the Problem Posing Research Project, we found that the nature of the employment of the photo established the relationship with the content of the problem. A problem was deemed as *interactive* if the photo was essential to complete the problem and coded *illustrative* if the photo was a visual enhancement or motivational device but unnecessary for solving the task. Note that careful observation of the photograph is essential for completion of the problems in Figure 4, rather than the photo being employed as an inspirational means. For example, students need to use the photo to respond to the question, "Describe the shapes in the photo, e.g. number of faces, edges. Categorise into 2D and 3D shapes." If the question was reposed as, "Describe the shapes you might find in a playground, e.g., number of faces, edges. Categorise into 2D and 3D shapes", the photograph would act as a catalyst for the problem and be considered illustrative rather than interactive. Therefore, while adopting illustrative questions may have educational merit, the central stimulus of the photo lacks function and purpose. The opportunity for building connections with the surrounding environment may be lost due to the insignificance of the photo in the problem solving process.

Employing open-ended problem pictures in the classroom

While working on one open-ended problem photo with a group of students, the image may be projected onto a large screen such as an interactive whiteboard. An interactive whiteboard is particularly useful for responding to questions such as "Name and draw the different shapes you can see in this photo." The students trace the outline of the shapes on the photo while it is projected onto the interactive whiteboard. The photo is hidden from the board to reveal only the traced outlines of the image. The same effect is achieved using an overhead transparency projector by placing a blank transparency over the photo transparency. Another alternative is to print photographs for small groups, partners or individuals to work with. Through the creation of open-ended questions with multiply entry points, the photos can be rotated between groups of students and the open-ended problem differentiate depending on the students' capabilities. This process allows for all students to have access to the task.

Students developing an eye for mathematics through photos

The process of creating open-ended problem photos can be extended to students through inviting the students to design their own photos and matching questions. The teacher might propose a mathematics topic e.g. fractions or 2D shape, and suggest students collect images that provoke this topic from their local environment, using digital cameras or mobile phone cameras, magazines or advertising material. The students could encourage their parents to help in the search for mathematical images. Creating a display of children's images outside the classroom may also encourage other classes to pose mathematics problems with personal meaning to them. The next step is inviting students to brainstorm questions they would like to solve from the photos. Creating opportunities that are real and relevant to the students provide possibilities to generate authentic engagement in mathematics (Bragg, Pullen, & Skinner, 2010). An exciting aspect of students developing open-ended problem photos is their awareness of the mathematics in everyday objects and their proactive approach to creating meaningful mathematical tasks.

It is important to include an instruction such as the following to the end of each open-ended problem photo: explain or illustrate your response. This prevents students from simply responding in an ad hoc manner with little consideration of the complexity of the problem. It is important to avoid the misconception that any response to an open-ended problem is acceptable. For example, when posed the question, "If this is a quarter of this playground, what are the possible perimeters of this playground?" a student may provide an answer of 100 m. It is difficult to determine whether this response is a guess or a thoughtfully considered response filled with mathematical complexity without further probing by the teacher. Requesting that students explain or illustrate their responses will provide the necessary evidence to determine the complexity of their cognitive processing of the problem.

Summary

Developing problems from photos not only provides opportunities to design open-ended problems but it also provides educators with a more critical mathematical lens through which to view mathematics. One preservice teacher's observation highlights this realisation, "I learned that math is really all around me, and that it is useful to me in everyday life, not just in school for homework from textbooks and tests from teachers." Collecting digital images and designing open-ended problems broadens teachers' awareness of grasping environmental opportunities for mathematics teaching and learning. As another preservice teacher confirmed, "…now I carry my digital camera around and have noticed more math in real life." Beware of the infectious nature of the 'math curse'.

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